HEAT TRANSFER IN LAMINAR CONVECTION COMBINED WITH INJECTION (SUCTION)

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An equation derived from an analysis of numerical results is presented for heat transfer in combined laminar convection.

The problem of heat transfer in laminar (free or forced) convection combined with injection (suction), which is specified by a series of equations for a uniform boundary layer, has been solved [1] using iterative methods.

Differential equations for free laminar boundary layers, neglecting viscous dissipation and assuming the physical parameters to be constant, are written for vertical porous surfaces in the form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,$$
$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty), \tag{1}$$
$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=a\frac{\partial^2 T}{\partial y^2}$$

with the following boundary conditions:

$$\begin{gathered} u=0,\quad v=v_w,\quad T=T_w \quad \text{for} \quad y=0; \\ u=U_\infty,\quad T=T_\infty \quad \text{for} \quad y=\infty. \end{gathered} \tag{2}$$

Combined convection is investigated for free and forced flows with the same direction for $T_W > T_\infty$.

To study the boundary layer characteristics of both free and forced convection, the mutual effects of the free convection on the forced flow and of the forced flow on the free flow must be considered. The value $A = Gr/Re^2$ is the main parameter for determining the effect of the boundary forces on the forced convection and the value $B = Re/(2Gr^{1/2})$ is used to determine the effect of the forced flow on the convection.

In Fig. 1 profiles for the nondimensional velocity f' and the temperature θ in boundary layers with combined convection are compared, using the data of Shevchik [2], who investigated two methods of solving the problem for nonporous surfaces: by expanding in a series the velocity and temperature functions, in terms of Gr/Re^2, when forced convection has the major role and in terms of $Re/Gr^{1/2}$ when free convection has the major role; $Gr = g\beta(T_W - T_\infty)x^3/4\nu^2$ and $Re = U_\infty x/\nu$ are the Grashof and Reynolds numbers.

For heat transfer described by the numerical analysis [1], generalized ratios were obtained (for Pr = 0.72) using correction factors for nonporous surfaces:

a) for determining the effect of free convection on the forced flow

$$\frac{Nu}{Nu_{for}}=(1-1.69f_{for})[1+(0.34+0.86f_{for})A^{0.5+0.2f_{for}}], \tag{3}$$

where $f_{for} = -v_W\sqrt{Re}/U_\infty$ is the injection (suction) parameter;

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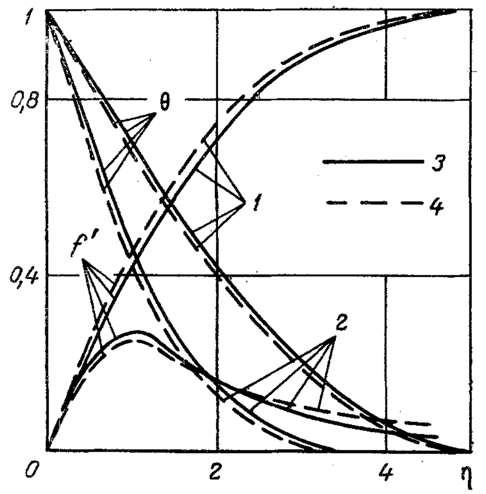

Fig. 1. Dimensionless velocity (f') and temperature (θ) profiles for Pr = 0.72; 1) A = 0.1; 2) B = 0.1; 3) solution according to Shevchik [2]; 4) the author.

b) for determining the effect of forced flow on the free convection

$$\frac{\mathrm{Nu}}{\mathrm{Nu}_{\mathrm{fr}}} = (1 - 0.77 f_{\mathrm{fr}})\,[1 + (0.24 + 0.02 f_{\mathrm{fr}})\, B^{1.16+0.8 f_{\mathrm{fr}}}], \tag{4}$$

where $f_{fr} = -\mathrm{Re}_w (\mathrm{Gr}/4)^{-1/4}$ is the injection (suction) parameter; $\mathrm{Re}_w = v_w x/\nu$.

In Eqs. (3) and (4) Nu_{for} and Nu_{fr} are the Nusselt numbers for forced and free flows in the absence of injection (suction).

Equation (3) is used to determine the heat transfer when the free convection affects the forced flow, and Eq. (4) is used when the forced flow affects the free flow. For flow conditions in which the inertia and bouyancy forces are comparable, another equation for the Nusselt number for nonporous surfaces was obtained by simple rearrangement of Eqs. (3) and (4):

$$\mathrm{Nu}_0^* = \begin{cases} 0.3\,\mathrm{Re}^{\frac{1}{2}} \left[1 + 0.34 A^{\frac{1}{2}}\right], & \text{if} \quad A < 9, \\ 0.504 \left(\dfrac{\mathrm{Gr}}{4}\right)^{\frac{1}{4}} [1 + 0.24 B^{1.16}], & \text{if} \quad A > 9. \end{cases} \tag{5}$$

Comparison of the values calculated using Eq. (5) with the experimental data and numerical results of Kligel [2] are presented in Fig. 2. The results presented were obtained using Eq. (3) (curve 1) and Eq. (4) (curve 2). To calculate the heat transfer in the vicinity of porous surfaces under conditions of combined convection, another equation was derived:

$$\mathrm{Nu}^* = \begin{cases} 0.3 a_1\,\mathrm{Re}^{\frac{1}{2}} [1 + b_1 A^{c_1}], & \text{if} \quad A < 9, \\ 0.504 a_2 \left(\dfrac{\mathrm{Gr}}{4}\right)^{\frac{1}{4}} [1 + b_2 B^{c_2}], & \text{if} \quad A > 9, \end{cases} \tag{6}$$

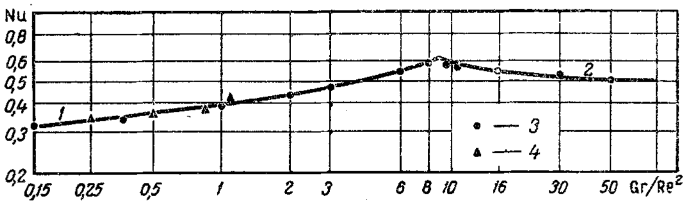

Fig. 2. Heat exchange in combined convection, Pr = 0.72: 1) according to Eq. (3) at $f_{for} = 0$; 2) according to Eq. (4) at $f_{fr} = 0$; 3) according to Eq. (5); 4) experimental data of Kligel [2].

where

$$a_1 = 1 - 1.69f_{\text{for}},\ a_2 = 1 - 0.77f_{\text{fr}};$$
$$b_1 = 0.34 + 0.86f_{\text{for}},\ b_2 = 0.24 + 0.02f_{\text{fr}};$$
$$c_1 = 0.5 + 0.2f_{\text{for}},\ c_2 = 1.16 + 0.8f_{\text{fr}};$$

In Eqs. (5) and (6), the parameters of the effect of free convection on the forced flow A and of the effect of the forced convection on the free flow B are related as follows

$$A = (2B)^{-2}.$$

Without injection (suction), i.e., when f_{for} and f_{fr} are zero, Eq. (6) takes the form of (5) for nonporous surfaces.

NOTATION

x, y are Cartesian coordinates;
u, v are projections of the velocity in the boundary layer onto the x, y axes;
v_w is the rate of injection (suction);
ν is the kinematic viscosity;
g is the acceleration due to gravity;
β is the coefficient of thermal expansion;
a is the thermal diffusivity;
T is the temperature;
U_∞ is the velocity of the external flow;
η is a dimensionless variable;
A is the parameter of the effect of free convection on forced flow;
B is the parameter of the effect of forced motion on free flow.

LITERATURE CITED

1. P. M. Brdlik and V. I. Dubovik, Pribory i Tekh. Éksperim., No. 2 (1969).
2. A. A. Shevchik, Teploperedacha, Ser. 6, No. 4 (1964).